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$U(1)$ lattice gauge theory and $N = 2$ supersymmetric Yang-Mills theory

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Abstract

We discuss the physics of four-dimensional compact $U(1)$ lattice gauge theory from the point of view of softly broken $N=2$ supersymmetric $SU(2)$ Yang-Mills theory. We provide arguments in favor of (pseudo-)critical mass exponents $1/3$, $5/11$ and $1/2$, in agreement with the values observed in the computer simulations. We also show that the J^{CP} assignment of some of the lowest lying states can be naturally explained.

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1 Introduction

Recent computer simulations indicate that compact $U(1)$ lattice gauge theory has a second order phase transition for a finite value of β and that the critical exponents associated with the transition are non-trivial [1, 2, 3, 4]. This is a remarkable situation since it, according to ordinary folklore, implies that there exists a non-trivial continuum field theory with these critical exponents. The lattice system itself is fairly simple and one can transform it to a dual gauge field coupled to a monopole field [5, 6]. The monopole field is a lattice artifact caused by the compactness of the $U(1)$ gauge group used on the lattice, in the same way as the topological defects of the XY -model in two dimensions are caused by the $O(2)$ symmetry. The XY phase transition is related to the “liberation” of these topological defects and is of infinite order. Associated with this transition we have an Euclidean quantum field theory, namely the sine-Gordon theory for a special value of the coupling constant. The $U(1)$ lattice gauge theory in three dimensions describes formally a gauge theory with lattice monopoles. This theory has no other fixed point than the gaussian one, where we recover the familiar three-dimensional electrodynamics and which has no monopoles at all. There is no way of taking the continuum limit in which the monopoles survive. However, there exists a three-dimensional non-Abelian $SU(2)$ gauge-Higgs theory, the Georgi-Glashow model, which has monopoles with finite action* and where the $SU(2)$ gauge theory is spontaneously broken to $U(1)$. The physics of this model, confinement of the $U(1)$ charge and a corresponding non-vanishing string tension and a massive dual photon, is qualitatively the same as in the three-dimensional compact $U(1)$ lattice gauge theory [5], and the lattice theory describes similar long distance physics as the continuum model, but it cannot be used to *define* in a rigorous way the full quantum field theory by approaching a fixed point.

In the case of compact $U(1)$ in four dimensions, the situation is, at least apparently, more like the the XY -model in two dimensions than like the the compact $U(1)$ theory in three dimensions: we have a critical point, a phase transition which may be of an order higher than one, and (measured) non-trivial exponents. However, in the case of the XY -model it was a non-trivial task to identify an underlying local quantum field theory, and in the case of the $U(1)$ lattice theory in four dimensions we have, frankly speaking, no obvious candidate at all. We can thus take a pragmatic attitude and simply ask whether there exists a continuum model field theory which has qualitatively the same features as observed for the $U(1)$ lattice theory. Such a question has a meaning even if it may turn out that the $U(1)$ -transition is a (weakly) *first order phase transition*†, since in this case the $U(1)$ lattice theory and the un-

*In fact, the monopoles act as instantons in the three-dimensional Euclidean theory.

†The history of the $U(1)$ phase transition is rather turbulent. First it was classified as a second order transition, with critical exponents fluctuating from mean field exponents to non-trivial exponents. Then hysteresis was discovered and extensive computer simulations pointed to

derlying continuum theory should still possess analogous long distance properties: a phase where the $U(1)$ charge is confined, where the corresponding string tension is non-zero and where a number of “gauge balls” are observed, and a Coulomb phase where the monopoles anti-screen. Approaching the phase transition the non-trivial (pseudo-)scaling of the lattice model should be present in the continuum model too, and finally, some universality features of the renormalized charge in the $U(1)$ -lattice theory [7, 8] should also be explained. It is clear that this is a non-trivial task for the continuum quantum theory, irrespectively of whether or not a genuine scaling limit can be defined for the lattice model.

An obvious candidate for a continuum field theory which may describe the same physics as the $U(1)$ lattice gauge theory is a softly broken $N = 2$ supersymmetric $SU(2)$ gauge theory. Before the soft breaking it describes at low energies a $U(1)$ theory which for certain values of the moduli of the theory consists of a light monopole hyper-multiplet interacting with a dual photon multiplet [9]. After breaking to $N=1$ supersymmetry it describes, at low energies, in the vicinity of the same value of the moduli, a $U(1)$ theory with a massive dual photon, non-zero string tension and a monopole condensate [9]. In order to make contact to the lattice $U(1)$ theory we have to induce further soft breaking of the $N=1$ supersymmetry. This is not under control to the same extent as the breaking from $N = 2$ to $N=1$. In the rest of this article we try to argue, mainly heuristically, that there exists a softly broken $N = 2$ supersymmetric theory which describes most of the features observed for the $U(1)$ lattice gauge theory, such that this lattice theory may play the same role relative to certain softly broken $N = 2$ supersymmetric gauge theories as compact three-dimensional $U(1)$ lattice gauge theory plays with respect to the Georgi-Glashow model.

The rest of the article is organized as follows: in section 2 we review shortly the results from the numerical simulations of the four-dimensional compact $U(1)$ lattice gauge theory. In section 3 we describe the soft breaking of the $N = 2$ supersymmetric gauge theory relevant for the phase transition between a confining $U(1)$ theory and the $U(1)$ theory in the Coulomb phase. In section 4 we try to match the physics of section 2 and 3. Finally, section 5 contains a discussion.

2 Compact $U(1)$ lattice gauge theory

The lattice theory is defined by the action

$$S = - \sum_{\square} \beta \cos \theta_{\square}, \quad (1)$$

a first order transition. Finally new computer simulations, either changing topology of space-time from toroidal to spherical [1] or adding by hand monopole-like terms [3], restored the second order transition, again with non-trivial exponents. See [2] for more references.

where the summation is over all plaquettes \square on the lattice and $\theta_\square \in [0, 2\pi)$ is the argument of the product of the $U(1)$ link variables around the plaquette \square . If we define $F_{\mu\nu}$ by $\theta_\square = a^2 e_0 F_{\mu\nu}$, where a is the lattice spacing and $\beta = 1/e_0^2$, the action (1) reduces in formal limit $a \rightarrow 0$ to the standard continuum expression $S = \frac{1}{4} \int d^4x F_{\mu\nu}^2$. It is possible to add additional terms to (1) to get an extended coupling constant space. Many of the computer simulations are performed in this extended coupling constant space. We refer to [2] for details.

Part of the physics of the $U(1)$ lattice gauge theory is well understood [5, 6]. It has a two phase structure. For large bare coupling constant e_0 the system exhibits confinement of electric charge, while it for small values of the bare coupling is in the Coulomb phase with a massless photon. As the coupling constant increases, the coupling to diluted topological excitations, which can be interpreted as (lattice) magnetic monopole loops, decreases, and these magnetic monopole loops unbind at the phase transition point, beyond which the (lattice) monopoles condense and cause confinement of the electric charge and a linear rising potential between static test charges by the dual Meissner effect. In the Coulomb phase the static charges interact via the Coulomb potential, and when the bare charge approaches the critical value e_0^c the monopole loops will renormalize (anti-screen) the charge according to

$$V_{coulomb}(r) = -\frac{\alpha_r}{r}, \quad \alpha_r \equiv \frac{e_r^2}{4\pi}, \quad (2)$$

where the relation between the renormalized and the bare charge has been conjectured to be [7, 8]

$$\alpha_r(\alpha_0) = \alpha_r^c - \text{const.} \left(1 - \frac{\alpha_0}{\alpha_0^c}\right)^\lambda, \quad (3)$$

with both α_r^c and λ universal. While λ appears as a standard critical exponent, it is more surprising that α_r^c should be universal. The last conjecture arose by analogy with the XY-model and (3) is consistent with present numerical evidence.

The recent numerical simulations have gone much further [2]. The masses of so-called “gauge balls” have been measured, and approaching the critical point from the confinement region it is found that the masses $m_j(\beta)$ and the square root of the string tension, $\sqrt{\sigma(\beta)}$ scale as

$$m_j(\beta) \sim c_j(\beta_c - \beta)^\nu, \quad \sqrt{\sigma(\beta)} \sim (\beta_c - \beta)^\nu, \quad \nu \simeq 0.35 \pm 0.03 \quad (4)$$

For most of the masses m_j there exists in addition definite spin, parity and charge conjugation assignment J^{PC} . We refer to [2] for details.

The only exception from the above scaling is a state $J^{PC} = 0^{++}$ which is observed to scale with Gaussian exponent:

$$m_{0^{++}} \sim (\beta_c - \beta)^{\nu_g}, \quad \nu_g \simeq 0.51 \pm 0.03 \quad (5)$$

Another series of recent simulations use the lattice action (1), but add by hand a lattice monopole term [3], such that

$$S_{mono} = - \sum_{\square} \beta \cos \theta_{\square} - \lambda \sum_{\rho, x} |M_{\rho, x}|, \quad (6)$$

where $M_{\rho, x} = \varepsilon_{\rho\sigma\mu\nu}(\bar{\theta}_{\mu\nu, x+\sigma} - \bar{\theta}_{\mu\nu, x})/4\pi$ and the physical flux $\bar{\theta}_{\mu\nu, x} \in [-\pi, \pi)$ is related to the plaquette angle $\theta_{\mu\nu, x} \in (-4\pi, 4\pi)$ by $\theta_{\mu\nu, x} = \bar{\theta}_{\mu\nu, x} + 2\pi n_{\mu\nu, x}$. According to [3], a sufficiently large fixed value of the coupling constant λ ensures a second order transition for a finite value $\beta_c(\lambda)$, and with an exponent ν_{mono} , which is not the mean field exponent (5) nor the non-trivial exponent in eq. (4). According to [3] one has

$$\nu_{mono} \simeq 0.44 \pm 0.02. \quad (7)$$

The question we ask is simply: is there any continuum quantum field theory which is compatible with (some of) the above mentioned lattice measurements ?

3 U(1) from Supersymmetric Yang-Mills theory

In this section we discuss briefly the Seiberg-Witten derivation of a low energy effective $U(1)$ theory containing monopoles from the $N=2$ supersymmetric $SU(2)$ theory [9], and how soft breaking and the addition of a certain superpotential can create a $U(1)$ confinement-deconfinement phase transition lying entirely in the $N=0$ sector, with physics resembling those outlined above for lattice $U(1)$.

3.1 The Seiberg-Witten Solution and its soft breaking

While the quantum fluctuations of $N=4$ supersymmetric Yang-Mills theory are trivial, those of $N=2$ supersymmetric Yang-Mills theory consist of an infinite series of instanton corrections as well as a one-loop contribution [10]. Thus $N=2$ supersymmetric Yang-Mills theory contains non-trivial physics, but the symmetry constrains the quantum fluctuations so much that the theory can be analyzed in considerable detail. The ground state is parameterized by an order parameter $u = \langle \text{Tr } \phi^2 \rangle$ corresponding to the breaking of $SU(2)$ to $U(1)$. For large values of u we have a standard scenario: at energy scales $\mu \gg \sqrt{u}$ all field theoretical degrees of freedom contribute to the β -function, which corresponds to the asymptotically free theory. For energies lower than \sqrt{u} only the $U(1)$ part of the theory is effective. In these considerations the dynamical confinement scale

$$\Lambda_{N=2}^4 = \mu^4 \exp(-8\pi^2/g(\mu)^2)$$

obtained by the one-loop perturbative calculation plays no role. The remarkable observation by Seiberg and Witten was that even when $u \leq \Lambda_{N=2}^2$, where one would naively expect that non-Abelian dynamics was important, the system remains in the

$U(1)$ Coulomb phase due to supersymmetric cancellations of non-Abelian quantum fluctuations. As u decreases the effective electric charge associated with the unbroken $U(1)$ part of $SU(2)$ increases, while the masses of the solitonic excitations which are present in the theory, will decrease. Dictated by monodromy properties of the so-called prepotential of the effective low energy Lagrangian, the monopoles become massless at a point $u \sim \Lambda_{N=2}^2$ where the effective electric charge has an infrared Landau pole and diverges. However, in the neighborhood of $u \sim \Lambda_{N=2}^2$, this strongly coupled theory has an effective Lagrangian description as a weakly coupled theory when expressed in terms of dual variables, namely a monopole hyper-multiplet and a dual photon vector multiplet. The perturbative coupling constant is now $g_D = 4\pi/g$ and the point where monopoles condensate corresponds to $g_D = 0$.

Another remarkable observation of Seiberg and Witten is that the breaking of $N=2$ to $N=1$ supersymmetry by adding a mass term superpotential will generate a mass gap, originating from a condensation of the monopoles. By the dual Meissner effect this theory confines the electric $U(1)$ charge at distances larger than the inverse $N=2$ symmetry breaking scale. In terms of the underlying microscopic theory it is believed that the reduction of symmetry from $N=2$ to $N=1$ allows excitations closer to generic non-supersymmetric “confinement excitations”, but that the soft breaking ensures that the theory is still close enough to the $N=2$ to remain an effective $U(1)$ theory[‡].

Thus we see that in the Seiberg-Witten scenario we can, by introducing the mass term superpotential, describe a $U(1)$ confining-deconfining transition. However, the transition is from the $N=2$ Coulomb phase to the $N=1$ confining phase, while we are interested in a $U(1)$ confining-deconfining transition occurring in the $N=0$ sector.

The breaking down to $N=0$ has been analyzed in a number of papers [11, 12], where soft breaking via spurion fields of $N=1$ and $N=2$ supersymmetric gauge theories are discussed (see also [13]). One of the motivations behind these studies has been to test if the Seiberg-Witten scenario can be extrapolated to realistic models for QCD confinement. Here we will use the same philosophy to test if it is possible to explain the lattice physics described in section 2 in terms of continuum physics of an extended model, precisely as the physics of compact lattice $U(1)$ in three dimensions is described by the continuum Georgi-Glashow model.

[‡]Or, as many people believe, that the generic confinement excitations *are* “monopole-like”, as described in the softly broken $N=2$ theory, and that the difference in the allowed fluctuations in ordinary QCD compared to softly broken $N=2$ are such that they do not modify the confinement mechanism. Whether this is true or not true will be irrelevant for our use of the Seiberg-Witten results.

3.2 The model

3.2.1 Unbroken $N = 2$

The starting point is the $N=2$ supersymmetric Yang-Mills theory. In $N=1$ superspace notation the bare Lagrangian is given by

$$\mathcal{L}_{bare} = \frac{2}{g^2} \int d^2\theta d^2\bar{\theta} \text{Tr} (\Phi^\dagger e^{-2V} \Phi e^{2V}) + \frac{1}{2g^2} \left(\int d^2\theta \text{Tr} W^\alpha W_\alpha + h.c. \right), \quad (8)$$

where all fields are in the fundamental representation, where g is the bare gauge coupling, and where the $N=1$ chiral multiplet[§] $\Phi = (\phi, \psi)$ and the $N=1$ vector multiplet $W_\alpha = (v_\mu, \lambda)$ constitute the $N=2$ vector multiplet in the Wess-Zumino gauge.

The elimination of the D -component in (8) produces the term $\text{Tr}([\phi, \phi^\dagger]^2)/g^2$ in the potential, which has a flat direction $\phi = a\sigma^3/2$ with an arbitrary complex number a . The order parameter is given by $u = \langle \text{Tr}(\phi^2) \rangle$. For generic values of u , the $SU(2)$ gauge symmetry breaks down to $U(1)$ and the low energy effective theory will be given by the Lagrangian of the $N=2$ $U(1)$ supersymmetric Yang-Mills theory

$$\mathcal{L}_{SW} = \frac{1}{4\pi} \text{Im} \left[\int d^2\theta d^2\bar{\theta} \frac{\partial \mathcal{F}}{\partial A} \bar{A} + \frac{1}{2} \int d^2\theta \frac{\partial^2 \mathcal{F}}{\partial A^2} W^\alpha W_\alpha \right], \quad (9)$$

where the prepotential \mathcal{F} is a holomorphic function of A (whose lowest component is a) and the dynamical scale $\Lambda_{N=2}$, generated by the non-Abelian interactions.

The functional form of the prepotential is uniquely determined from the monodromy properties of the singularities expected from duality and the spectrum of solitonic states [9, 14]. In the $SU(2)$ case, there are three singularities. One is the semi-classical one with a at infinity, and the others at, say $u = \pm \Lambda_{N=2}^2$, where a monopole or a dyon becomes massless, respectively. Near the point $u = \Lambda_{N=2}^2$ where the monopoles become massless and the theory is strongly coupled in terms of the original field variables, a duality transformation results in a weakly coupled effective Lagrangian of the monopole field

$$\mathcal{L}_M = \int d^2\theta d^2\bar{\theta} (M^\star e^{2V_D} M + \tilde{M}^\star e^{-2V_D} \tilde{M}) + \left(\int d^2\theta \sqrt{2} A_D M \tilde{M} + h.c. \right), \quad (10)$$

where (M, \tilde{M}) is the monopole hyper-multiplet and where A_D and V_D represent the $N=2$ vector multiplet of the dual photon.

As mentioned, Seiberg and Witten introduced a breaking of the $N=2$ supersymmetry to $N=1$ by adding a superpotential bU , $U = \text{Tr} \Phi^2$, for the $N=1$ chiral multiplet. Adding this piece breaks the flatness of the scalar potential and u is no

[§]Our superfield notation is as follows: $\Phi = a_\Phi + \sqrt{2}\theta\psi_\Phi + \theta\theta f_\Phi$ with the F -term being the coefficient to $\theta\theta$, $\Phi|_F \equiv f_\Phi$, the A -term the coefficient to 1, $\Phi|_A \equiv a_\Phi$, and the D -term the coefficient to $\theta^2\bar{\theta}^2$, i.e. for instance $\Phi^\dagger\Phi|_D \equiv |f_\Phi|^2$, etc.

longer a free parameter. As long as the breaking parameter b is suitably small, by comparison to $\Lambda_{N=2}$, the total superpotential is obtained as the sum of bU and the potential from (10), i.e.

$$W = \sqrt{2}A_D M \tilde{M} + bU(A_D). \quad (11)$$

When the breaking parameter $b \neq 0$, the vacuum, defined by $dW = 0$, satisfies

$$\begin{aligned} \sqrt{2}m\tilde{m} + b\frac{du}{da_D} &= 0, \\ a_D m &= a_D \tilde{m} = 0, \end{aligned} \quad (12)$$

where m , \tilde{m} and a_D denote the corresponding scalar components of M , \tilde{M} and A_D , respectively. The solution is

$$\begin{aligned} m &= \tilde{m} = \sqrt{-bu'(0)/\sqrt{2}} \sim \sqrt{b\Lambda_{N=2}}, \\ a_D &= 0. \end{aligned} \quad (13)$$

The interpretation of (13) is that a monopole condensate is formed and that the potential generated from the superpotential (11) in this respect behaves like an ordinary Ginzburg-Landau potential.

3.2.2 Soft breaking by spurions

A more general scheme of soft breaking of the Seiberg-Witten solution, still respecting the monodromy properties of the singularities, was obtained in the references [12]. They introduced a spurion $N=2$ vector multiplet, the dilaton spurion, and the dynamical scale[¶] Λ is expressed as $\exp(is)$, where s denotes the lowest scalar component of the dilaton spurion S . Thus the effective Lagrangian of the softly broken $N=2$ supersymmetric Yang-Mills theory is given by

$$\begin{aligned} \mathcal{L}_{soft} &= \frac{1}{4\pi} \text{Im} \left[\int d^2\theta d^2\bar{\theta} \frac{\partial \mathcal{F}}{\partial A^i} \bar{A}^i + \frac{1}{2} \int d^2\theta \frac{\partial^2 \mathcal{F}}{\partial A^i \partial A^j} W^{i\alpha} W^j_{\alpha} \right], \\ i &= 0, 1; \quad A^0 = S, \quad A^1 = A, \end{aligned} \quad (14)$$

where the lowest component and the auxiliary fields of the spurion fields A^0 and W_{α}^0 are frozen to constant values, thus breaking the $N=2$ directly down to $N=0$.

In reference [12], the softly broken model with $\mathcal{L}_{soft} + \mathcal{L}_M$ given by (10) and (14) is analyzed. Since the monopoles have already been included in the heavy modes which are integrated out in \mathcal{L}_{SW} , the authors of [12] argue that the monopole field in (10) should be understood as representing the classical monopole field. This softly

[¶]In the rest of this section we drop the suffix $N=2$ on Λ in order to avoid too cumbersome a notation.

broken model was shown to be in the confinement phase, in the same way as the original $N=1$ model of Seiberg and Witten, the dynamics of the confinement being monitored by a monopole condensation dictated by the freezing of S (see [12] for details). In the approach of these authors, the parameter F_0 , which controls the breaking down to $N=0$, plays the same role as the parameter b in the approach of Seiberg and Witten, and has the same undesirable (for our purposes of comparing with compact $U(1)$ on the lattice) feature that there is no $N=0$ Coulomb phase.

3.2.3 Monitoring confinement-deconfinement

In order to create a scenario which is closer to the confinement-deconfinement transition observed on the lattice we need to have an $N=0$ theory on both sides of the confining transition.

To achieve this purpose we add an additional $N = 1$ Lagrangian

$$\mathcal{L}_z = \int d^2\theta d^2\bar{\theta} z^\dagger z + \left(\int d^2\theta l z (w - \text{Tr}(\Phi^2)) + h.c. \right). \quad (15)$$

to the original Lagrangian (8). Here z is an $N=1$ chiral multiplet without any gauge charges, and l and w are free complex parameters. In the $l \rightarrow 0$ limit, z decouples from the original system and will go back to the model mentioned in the previous subsection. In the $l \rightarrow \infty$ limit, the kinetic term of z is negligible, and z is an auxiliary field.

Consider the $N=1$ case of $\mathcal{L}_{bare} + \mathcal{L}_z$ in (8) and (15). After elimination of the D and F components, we obtain the potential

$$V_{bare+z} = |l|^2 |w - \text{Tr} \phi^2|^2 + 4g^2 |lz|^2 \text{Tr}(\phi^\dagger \phi) + \frac{1}{g^2} \text{Tr}([\phi, \phi^\dagger]^2). \quad (16)$$

The first term of (16) will constrain the value of $\text{Tr} \phi^2$ to be close to w ; when w is very large the system can be treated semi-classically. Thus the gauge group will be broken at the scale of order \sqrt{w} and the system will be in the $U(1)$ Coulomb phase. Taking the value w smaller, the effective coupling of the system becomes stronger, but precisely as for the original model (8), the holomorphy argument [15] ensures that the system will stay in the $U(1)$ Coulomb phase. On the other hand, if the soft-breaking terms are introduced and the supersymmetry is broken, the delicate cancellation will be lost and the system will naturally be in the confinement phase for small w .

Now consider the effect of the addition of \mathcal{L}_z in the dual picture. This system is naively expected to be described by the effective Lagrangian $\mathcal{L}_{SW}^D + \mathcal{L}_M + \mathcal{L}_z$, where \mathcal{L}_{SW}^D is the counterpart of \mathcal{L}_{SW} in the dual variables. But, since this is the effective low energy lagrangian after the breakdown from $SU(2)$ to $U(1)$ and \mathcal{L}_z respects only $N = 1$ supersymmetry, some quantum corrections can appear. To find the possible quantum corrections to the superpotential, consider the global

symmetry $U(1) \times U(1)_R$ of this system: $W^\alpha W_\alpha = (0, -2)$, $A = (1, 0)$. The charges of the fields in the dual picture and the couplings are given by

$$\begin{aligned} W_D^\alpha W_{D\alpha} &= (0, -2), \quad M\tilde{M} = (-1, -2), \quad A_D = (1, 0), \\ \Lambda &= (1, 0), \quad zl = (-2, -2), \quad w = (2, 0). \end{aligned} \quad (17)$$

The first $U(1)$ is anomalous, but the anomaly is cancelled by assigning a charge to the dynamical scale Λ [15] as in the case where \mathcal{L}_z is absent. Based on the charge assignments (17) the corrections to the superpotential conserving the charges must have the form

$$W_{q.c.} = (W_D^\alpha W_{D\alpha})^{1+n_1} (M\tilde{M})^{n_2} (zl)^{-n_1-n_2} \Lambda^{-2n_1-n_2} f\left(\frac{A_D^2}{w}, \frac{\Lambda^2}{w}\right) \quad (18)$$

with integers $n_1 \geq -1$ and $n_2 \geq 0$ and a two-parameter function f . In this derivation we assumed that the generated superpotential is regular at $W_D^\alpha, M, \tilde{M} \sim 0$. In the $l \rightarrow 0$ limit, z decouples and $W_{q.c.}$ should vanish. This requirement determines that $n_1 = -1$ and $n_2 = 0$. Moreover the parameter Λ would appear like Λ^4 in the instanton corrections to the superpotential. Thus we can parameterize the quantum corrections to the superpotential as

$$W_{q.c.} = \frac{\Lambda^4 zl}{w} f\left(\frac{A_D^2}{w}, \frac{\Lambda^4}{w^2}\right), \quad (19)$$

and the effective Lagrangian in the dual description will be given by

$$\mathcal{L}_{SW}^D + \mathcal{L}_M + \int d^2\theta d^2\bar{\theta} K(z, z^\dagger) + \left(\int d^2\theta lz \left\{ w - U(A_D) + \frac{\Lambda^4}{w} f\left(\frac{A_D^2}{w}, \frac{\Lambda^4}{w^2}\right) \right\} + h.c. \right). \quad (20)$$

The elimination of the F -component of z gives a potential term

$$K_{zz^\dagger}^{-1} |l|^2 \left| w - u(a_D) + \frac{\Lambda^4}{w} f\left(\frac{a_D^2}{w}, \frac{\Lambda^4}{w^2}\right) \right|^2. \quad (21)$$

This term will of course complicate further analysis due to the unknown function f and the unknown Kähler potential K . We will *assume* that the auxiliary field limit of z , i.e. the limit $l \rightarrow \infty$, simplifies the analysis in such a way that the potential (21) simply provides a constraint which relates the free parameter w and a_D . Since the parameter w does not appear in the other terms of the whole potential unless we consider higher derivative terms of the effective Lagrangian, we can regard a_D as a free parameter instead of w , and ignore the parameter w in the following discussions.

3.2.4 The vacuum structure

We now analyze the vacuum structure of the softly broken model given by the Lagrangian (20) with substitution of \mathcal{L}_{SW}^D with \mathcal{L}_{soft}^D , following the analysis in

[12]. Then, after the eliminations of the D and F components of the dual photon and the monopole fields, we obtain the potential

$$V_{soft+z} = \frac{1}{b_{11}} \left| b_{01} \bar{F}_0 + \sqrt{2} m \tilde{m} + z l \frac{\partial u_q}{\partial a_D^1} \right|^2 + \frac{1}{2b_{11}} (b_{01} D_0 + |m|^2 - |\tilde{m}|^2)^2 \\ + 2|a_D|^2 (|m|^2 + |\tilde{m}|^2) - b_{00} |F_0|^2 - \frac{b_{00}}{2} D_0^2 + \left(F_0 z l \frac{\partial u_q}{\partial a_D^0} + h.c. \right), \quad (22)$$

where we ignored the potential (21) for the reason mentioned above. In (22) (m, \tilde{m}) denote the scalar components of the monopole hyper-multiplet (M, \tilde{M}) and

$$u_q = u - \frac{\Lambda^4}{w} f \left(\frac{a_D^2}{w}, \frac{\Lambda^4}{w^2} \right), \quad (23)$$

while

$$b_{ij} \equiv \frac{1}{4\pi} \text{Im } \tau_{ij} = \frac{1}{4\pi} \text{Im } \frac{\partial^2 \mathcal{F}}{\partial a_D^i \partial a_D^j}. \quad (24)$$

Finally F_0 and D_0 denote the frozen F and D components of the spurion multiplet, respectively.

In [12] the choice of parameters

$$F_0 \lesssim \Lambda, \quad D_0 = 0 \quad (25)$$

was studied. In our case we find that this choice leads to an unbounded potential V_{F_0} in a neighborhood of $a_D = 0$. This implies that we have to take into account the contributions to the potential from higher derivative terms in order to understand the dynamics in this region. Since we have little control over these higher derivative terms we will not consider the case (25) any further and turn to

$$F_0 = 0, \quad D_0 \neq 0. \quad (26)$$

From (22) we obtain, setting $F_0 = 0$,

$$V_{D_0} = \frac{1}{b_{11}} \left| \sqrt{2} m \tilde{m} + z l \frac{\partial u_q}{\partial a_D^1} \right|^2 + \frac{1}{2b_{11}} (b_{01} D_0 + |m|^2 - |\tilde{m}|^2)^2 \\ + 2|a_D|^2 (|m|^2 + |\tilde{m}|^2) - \frac{b_{00}}{2} D_0^2. \quad (27)$$

This potential has obviously a minimum with respect to the field z , and we use this value for z . Taking derivatives with respect to the monopole fields, we obtain

$$\frac{\partial V_{D_0}}{\partial m} = \left(\frac{1}{b_{11}} (b_{01} D_0 + |m|^2 - |\tilde{m}|^2) + 2|a_D|^2 \right) m = 0, \\ \frac{\partial V_{D_0}}{\partial \tilde{m}} = \left(-\frac{1}{b_{11}} (b_{01} D_0 + |m|^2 - |\tilde{m}|^2) + 2|a_D|^2 \right) \tilde{m} = 0. \quad (28)$$

Thus if

$$|a_D|^2 < \frac{|b_{01}D_0|}{2b_{11}}, \quad (29)$$

these equations (28) have solutions other than the trivial $m = \tilde{m} = 0$ ^{||}. They are

$$\begin{aligned} |\tilde{m}|^2 &= -2b_{11}|a_D|^2 + b_{01}D_0, & m = 0 & \text{ for } b_{01}D_0 > 0, \\ |m|^2 &= -2b_{11}|a_D|^2 - b_{01}D_0, & \tilde{m} = 0 & \text{ for } b_{01}D_0 < 0. \end{aligned} \quad (30)$$

In both cases, $z = 0$. One can easily show that these non-trivial solutions correspond to the absolute minima. The non-zero vacuum expectation values of the monopole fields give a mass to the dual photon. Following the general folklore this leads to a confined electric charge by the dual Meissner effect. Outside the region (29), the monopole fields do not condensate, and the system has to be in the Coulomb phase. Thus we have shown that by adding a term (15) there may be a confinement-deconfinement phase transition line in the parameter space of a_D (or w), *while the system on both sides of the phase transition line can be considered as an effective $N=0$ theory.*

To regard this phase transition line as that of the pure compact $U(1)$, it is important that no new massless degrees of freedom appear at the transition. As shown previously the system has a non-gauge global $U(1) \times U(1)_R$ symmetry^{**}. The $U(1)$ symmetry is anomalous, and hence is irrelevant here. The $U(1)_R$ symmetry might be broken in the Higgs phase of the dual description and thus generate zero-mass particles by the Goldstone mechanism, but actually $U(1)_R$ is not broken because $m\tilde{m} = z = 0$ ^{††} for the solutions (30). Hence, near the phase transition line, the light degrees of freedom are the dual photon and the one of the monopole fields which becomes massless at the phase transition line by the Landau-Ginzburg mechanism.

In the next section we will analyze the phenomenology close to this phase transition line based on the picture of the dual photon and the monopole.

4 Phenomenology

4.1 Confinement phase

In the confinement phase, close to the phase boundary between the confinement and the deconfinement phase, we have an effective Ginzburg-Landau description in terms of monopole fields and a dual photon as in (11) and (13). Although the situation in (27) and (30) is slightly more complicated than in the simplest breaking to $N=1$

^{||}If $D_0 = 0$, we have $N=1$ and in addition there is no region of monopole condensation. This is consistent with the holomorphy argument given below eq. (16).

^{**}The breaking parameter D_0 has the $U(1) \times U(1)_R$ charge $D_0 = (0, 0)$.

^{††}Considering m and \tilde{m} separately, one may conclude there is a broken symmetry. But this is the dual $U(1)$ gauge symmetry and does not generate massless particles by the Higgs mechanism.

described in (13) the order of magnitude is the same for the parameters involved, and we will in the following use the simplified notation of a breaking parameter b which monitors the distance from the points in coupling constant space where monopoles condense. In the original picture of Seiberg and Witten it corresponds to the point where $N=2$ is broken, which at the same time corresponds to the confinement-deconfinement transition of the $U(1)$ charge. In the present generalized model b corresponds to the distance to the confinement-deconfinement transition. In both cases we have (as in (13))

$$m \sim \sqrt{b\Lambda_{N=2}}. \quad (31)$$

In the more elaborate model considered above, the breaking from $N=2$ to $N=0$ is partly separated from the confinement-deconfinement transition. The supersymmetry breaking occurs at a scale D_0 , while monopole condensation is dictated by (29). Thus b is here a function of D_0 and a_D (or w), as will be discussed below.

Let us now relate the scale b to the non-perturbative physical scale in the confinement phase. Consider first the simplest case where $N=2$ is broken to $N=1$. The running coupling constant is given by

$$\frac{1}{g_{N=2}^2(\mu)} = \frac{4}{8\pi^2} \log \frac{\mu}{\Lambda_{N=2}}. \quad (32)$$

Now assume that the breaking scale b is larger than the dynamical scale $\Lambda_{N=2}$. When the theory is broken to $N=1$, some of the components become massive and will not contribute to the further running of the coupling constant. For the light $N=1$ components we have

$$\frac{1}{g_{N=1}^2(\mu)} = \frac{6}{8\pi^2} \log \frac{\mu}{\Lambda_{N=1}}. \quad (33)$$

The scales of the theories can be related by the matching condition $g_{N=1}^2(b) = g_{N=2}^2(b)$ at the breaking scale, which, according to standard arguments, implies [16]

$$\Lambda_{N=1}^6 = b^2 \Lambda_{N=2}^4. \quad (34)$$

We can repeat the argument in the case where the breaking is all the way from $N=2$ down to $N=0$. Now the parameter b includes two free parameters D_0 and a_D (or w) as can be seen from (30) and (31): $b = (|b_{01}D_0| - 2b_{11}|a_D|^2)/\Lambda_{N=2}$. Since D_0 is the breaking scale of $N=2$ supersymmetry, it has a clearer meaning in the original system than the parameter a_D (or w) which is linked to the unknown function f and the limit $l \rightarrow \infty$ in (21). Thus, in the following discussion, for the purpose of clarity let us leave the parameter a_D (or w) fixed and regard the D_0 as the free parameter which monitors the phase transition. Assume now that the parameter D_0 is larger than the dynamical scale $\Lambda_{N=2}$. Then comparing (30) and (31), the parameter b may be identified to the breaking scale D_0 , since $b_{01} \sim \Lambda_{N=2}$ and a_D is considered

fixed. In this case the fermionic partner to the gauge field becomes massive below the breaking scale and (33) is replaced by

$$\frac{1}{g_{N=0}^2(\mu)} = \frac{11}{12\pi^2} \log \frac{\mu}{\Lambda_{N=0}}, \quad (35)$$

from which we conclude that

$$\Lambda_{N=0}^{\frac{22}{3}} = b^{\frac{10}{3}} \Lambda_{N=2}^4. \quad (36)$$

The non-perturbative scales $\Lambda_{N=1}$ and $\Lambda_{N=0}$, respectively, are the scales in which any mass (string tension, gauge-balls, etc.) should be measured in the confinement regime of the $N=1$ and the $N=0$ theories.

4.1.1 Mapping on compact lattice $U(1)$

If we want to map the physics of the $U(1)$ part of the broken $N=2$ supersymmetric theory onto the compact $U(1)$ lattice theory it is natural to identify the symmetry breaking parameter b with $\beta_c - \beta$, since one in this case gets a Ginzburg-Landau expression for the monopole condensate from (31) and the monopole condensate forms on the lattice when $\beta_c - \beta$ becomes positive and it forms in the continuum model when $b > 0$. Under this *assumption*, plus the *assumption* that the qualitative physics of the $U(1)$ sector is the same in the continuum theory and the lattice theory (like in three dimensions), we get for the non-perturbative mass scales $\Lambda(\beta)$ in the confinement sector:

$$\Lambda(\beta) \sim (\beta_c - \beta)^{\frac{1}{3}} \quad \text{for } N=2 \rightarrow N=1, \quad (37)$$

$$\Lambda(\beta) \sim (\beta_c - \beta)^{\frac{5}{11}} \quad \text{for } N=2 \rightarrow N=0. \quad (38)$$

These exponents are just the ones measured in the lattice simulations of various compact $U(1)$ theories, as discussed in sec. 2. Further, the above scenario also explains why there should be excitations in the system associated with the mean field Ginzburg-Landau exponents

$$M(\beta) \sim (\beta_c - \beta)^{\frac{1}{2}} \quad (39)$$

for excitations of the monopole condensate.

We do not pretend that the arguments above are conclusive. Strictly speaking the matching of scales by the one loop formula is only valid if b is large, and we want to use the matching in a region where it is not the case. Further, the breaking to $N=1$ is of course an incomplete breaking, and if the mass exponent $1/3$ should be compared to lattice results, further breaking down to $N=0$ should be implemented in a way not affecting the scaling. We have no explicit suggestions to how this

could occur. In addition the actual value of the renormalized charge fine-structure constant α in the lattice simulations is $\alpha \approx 0.2$, i.e. $\alpha_D \approx 5$. This is not exactly in the region where the dual photon-monopole picture is believed to be reliable. On the other hand the direct translation of the lattice charge α to the continuum is not clear, so these values might be misleading from a continuum point of view. Finally, the occurrence of a non-perturbative scaling and a mean field scaling at the same time is a somewhat unusual situation. Nevertheless, it is precisely what *is* observed in the lattice simulations. From the above considerations the physics of the two different lattice systems studied ($U(1)$ without an explicit monopole term in the action, and $U(1)$ with a monopole term included) should then be identified with the physics of different softly broken $N=2$ supersymmetric theories and this identification explains to some extent the observed non-trivial critical exponents ($\nu = 0.35 \pm 0.03$ and $\nu = 0.44 \pm 0.02$) and in addition the mean field exponents which are also observed.

4.1.2 The one-particle spectrum

In order to substantiate further that the physics of compact lattice $U(1)$ is the same as broken $N=2$ supersymmetric Yang-Mills theory we discuss below the parity and charge conjugation quantum numbers of the one-particle state in the confinement phase of the broken supersymmetric theory and compare it with the measurements on the lattice.

The pure $U(1)$ gauge theory $\int d^4x F_{\mu\nu} F^{\mu\nu}$ is obviously invariant under the parity and charge conjugation transformation defined by

$$\begin{aligned} P &: A_0 \rightarrow A_0, \quad A_i \rightarrow -A_i, \\ C &: A_\mu \rightarrow -A_\mu, \end{aligned} \tag{40}$$

where A_μ are the gauge potential. Under these transformations the field strengths transform as

$$\begin{aligned} P &: E \rightarrow -E, \quad B \rightarrow B, \\ C &: E, B \rightarrow -E, -B. \end{aligned} \tag{41}$$

Defining the duality transformation by $E^D = B$ and $B^D = E$, one obtains the parity and charge conjugation transformation of the dual gauge field as

$$\begin{aligned} P &: A_0^D \rightarrow -A_0^D, \quad A_i^D \rightarrow A_i^D, \\ C &: A_\mu^D \rightarrow -A_\mu^D, \end{aligned} \tag{42}$$

Thus the dual photon field is 1^{+-} , where J^{PC} denotes a spin J , parity P and charge conjugation C state. Since the covariant derivative $D_\mu^D \equiv \partial_\mu + iA_\mu^D$ transforms as

$$\begin{aligned} P &: D_0^D \rightarrow D_0^{D*}, \quad D_i^D \rightarrow -D_i^{D*}, \\ C &: D_\mu^D \rightarrow D_\mu^{D*}, \end{aligned} \tag{43}$$

the transformations of the monopole field M are given by

$$\begin{aligned} P &: M \rightarrow \eta_P M^*, \\ C &: M \rightarrow \eta_C M^*, \end{aligned} \tag{44}$$

where $\eta_{P,C}$ are the phase factor ambiguities coming from the global $U(1)$ dual gauge symmetry of the monopole field action. To fix this ambiguity, we take the vacuum expectation value v_M of the monopole field to be real by using the global gauge transformation. Then the appropriate P and C transformations which preserve the vacuum expectation value are given by just taking $\eta_{P,C} = 1$.[‡] Thus one concludes that the physical monopole field, $M_R \equiv \text{real part of } M$, has the quantum number 0^{++} .

This quantum number assignment is quite interesting. The lattice spectroscopy shows that there are the one-particle states with quantum numbers 0^{++} and 1^{+-} in the confinement phase [2]. We identify these states as the monopole (or, in the case where a monopole condensate is formed, as the lowest excitation in this condensate) and the dual photon one-particle state, respectively. We see that this assignment is in accordance with the identification of the monopole condensate excitations as the ones with the mean field exponents, since the 0^{++} state has $\nu = 1/2$.

4.2 Coulomb phase

In the broken supersymmetric model we move into the Coulomb phase when a_D is sufficiently large. Deep into this phase the low energy fields excitations involve just the ordinary photon field. The monopoles will be very heavy. This is trivially in agreement with the lattice picture. As mentioned in sec. 2 the physics of compact lattice $U(1)$ is quite interesting near the phase transition: There seems to be a universal value of the “renormalized” charge in the theory, caused by the anti-screening of the vacuum monopole fluctuations. From the philosophy used so far, one could hope that it was possible to predict the value α_R^c , as well as the critical exponent λ in the expression (3). We have not yet made any progress in that direction.

5 Summary and discussions

The recent remarkable lattice simulations of compact $U(1)$ show very interesting physics with a confinement-deconfinement transition which is reported to be of second order with non-trivial critical mass exponents.

[‡]In fact one can continue the discussion without taking special values of $\eta_{P,C}$. In this case one must consider the gauge invariant field $M_R \equiv v_M^* M$ instead of just $M_R \equiv \text{real part of } M$ as the physical monopole field. For simplicity, we take the special values.

If the transition is second order, we have a new and very interesting situation in quantum field theory since there should be an underlying continuum field theory of a new kind. Presently we have nothing more to say about such a revolutionary scenario.

In this paper we have tried something more modest, namely to find *a* continuum, four-dimensional field theory which at a qualitative level has the same low energy physics as the lattice model and thus might explain the observed critical exponents. Thus this continuum model will retain its explanatory value even if the transition is not really a second order transition, but a weakly first order transition which has only pseudo-critical exponents.

The XY-model and the sine-Gordon model (at a particular value of the coupling constant) constitute an example of the first kind: a non-trivial scaling limit of the XY-model exists and the physics in this limit is identical to that of the sine-Gordon theory at a special value of the coupling constant. Three-dimensional compact lattice $U(1)$ and the Georgi-Glashow model provides an example of the second kind: the physics is qualitatively of the same nature for low energy excitations, but no genuine scaling limit can be defined for the lattice theory, such that it becomes identical to the Georgi-Glashow model. According to folklore (which might now be falsified by the $U(1)$ lattice models) the supersymmetric field theories are the only four-dimensional field theories containing scalar fields, which have chance to exist as non-trivial, genuine interacting field theories. We have analyzed the soft breaking of a class of such continuum theories and have shown that the non-trivial dynamics of these theories indeed might explain the observed critical or pseudo-critical exponents, as well as the discrete quantum numbers assigned to some of the particles in the lattice theory.

If one assumes our heuristic arguments are correct, there is a chance that one in the future will be able to understand why the renormalized lattice charge in the Coulomb phase seems to converge to a universal number independent of the details of lattice implementation of the theory for $\beta \rightarrow \beta_c$.

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